

First Prairie DM Workshop Program - Abstracts

University of Regina

October 3-5, 2003

Local Organizers:

Brian Alspach and Shaun Fallat

Invited Speakers

Wolf Holzmann, University of Lethbridge

Don Kreher, Michigan Technological University

Xiangwen Li, University of Regina

Vazz Linek, University of Winnipeg

John Moon, University of Alberta

Joy Morris, University of Lethbridge

Ortrud Oellermann, University of Winnipeg

Bill Sands, University of Calgary

Ralph Stanton University of Manitoba

Funding

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Department of Mathematics and Statistics and from the
Faculty of Science at the
University of Regina

Location

Room 623, Education Bldg., University of Regina

1 Meeting Program (Room 623, Education Bldg.)

Friday, October 3, 2002

- 15:00-15:30 **Welcome & Coffee**
- 15:30-16:30 **Xiangwen Li**, *Group Coloring*
- 16:30-17:30 **Ralph Stanton**, *Discussion of g_4^4*
- 18:00- **Dinner** *India House Restuarant (directions on the Friday)*
- 20:00- **After Dinner Party** *Alspach's House (directions on the Friday)*

Saturday, October 4, 2003

- 10:00-11:00 **Joy Morris**, *Some results on wreath products of graphs*
- 11:00-11:15 **Break**
- 11:15-12:15 **Vazz Linek**, *Reachability Problems in Arc-colored Tournaments*
- 12:15-13:30 **Lunch**
- 13:30-14:30 **Wolf Holzmann**, *Plotkin Arrays*
- 14:30-15:30 **Ortrud Oellermann**, *Contour Vertices and Their Applications to Convexity Notions in Graphs*
- 15:30-16:00 **Break**
- 16:00-17:00 **John Moon**, *On Efficient Dominating Sets in Simply Generated Trees*
- 18:00- **Dinner** *TBA*

Sunday, October 5, 2003

10:00-11:00	Don Kreher , <i>Transverse t-designs</i>
11:00-11:15	Break
11:15-12:15	Bill Sands , <i>Two summer undergrads and their research</i>
12:15-	Closing remarks

2 Abstracts (alphabetical by speaker)

Plotkin Arrays

Wolf H. Holzmann

Plotkin introduced an orthogonal design $OD(24; 3, 3, 3, 3, 3, 3, 3, 3, 3)$ for the first time in 1971. It was consequently conjectured that all orthogonal designs

$$OD(8n; n, n, n, n, n, n, n, n)$$

exists for every positive integer n . The present status of this conjecture will be discussed in this talk.

Transverse t -designs

Donald L. Kreher

Given a partition H of the point set X into holes a subset is transverse if it intersects each of the holes in 0 or 1 point. A transverse t -design with respect to the partition H is a collection of k -element transverse subsets that cover each transverse t -element subset the same number of times. A transverse Steiner quadruple systems, (SQS) is a transverse 3-design with $k = 4$ in which each transverse 3-element set is covered exactly once. We

present a computational method for constructing transverse t -designs and use this method is to investigate transverse SQSs. We also develop direct and recursive constructions for transverse SQSs and provide a table of results on the small systems. (This is joint work with K.A. Lauinger, R. Rees, and D.R. Stinson)

Group Coloring

Xiangwen Li

The concept of group coloring was introduced by Jaeger, Linial, Payan and Tarsi in 1992. This talk will be a survey on the chromatic number of planar graphs, bipartite graphs and join of graphs. We also outline some results on the group coloring version of Brook's Theorem and list some open problems.

Reachability Problems in Arc-colored Tournaments

Václav Linek

In 1982 Sands, Sauer and Woodrow proved a theorem with the following corollary: If the arcs of a tournament are 2-colored, then there is a vertex v (a *sink*) in the tournament such that any other vertex in the tournament has a monochromatic directed path into v . Since then this corollary has been generalized and similar results have appeared. Here we investigate a natural generalization, where the colors on the arcs of the tournament are the *vertices* of a directed graph, A , and a directed walk in the tournament is an A -walk iff the colors on the arcs of the walk in the tournament also form a walk in A . For example, if A has just two vertices and each vertex has a loop to itself, then the A -walks/ A -paths are exactly the directed monochromatic walks/paths. We call A 1-sinkable if any arc coloring of any tournament has a sink that can be reached from any other vertex by an A -walk; so the result of Sands et. al. says that the digraph of two loops is 1-sinkable. We will give examples of other 1-sinkable digraphs, and a striking counter-example that refutes an obvious conjecture characterizing the property of 1-sinkability. We shall also consider two natural refinements of the class of 1-sinkable digraphs and progress on the classification problem for these refined classes.

On Efficient Dominating Sets in Simply Generated Trees

John W. Moon

A subset S of the nodes of an oriented tree T^* is an efficient dominating set (EDS) if (i) no node of S dominates any other node of S and (ii) each node not in S is dominated by exactly one node of S . (See, e.g., Bange, Barkauskas, and Clark, *Ars Comb.* 45 (1997) 201-207.) Any oriented tree has at most one EDS. An orientation of the edges of an unoriented tree T is admissible if the corresponding oriented tree T^* has an EFS. We investigate $e(n)$, the average number of admissible orientations of trees with n nodes that belong to simply generated families of rooted trees, and $m(n)$, the average size of the EDS's of the oriented trees obtained from these trees by admissible orientations. If mild conditions are satisfied then the n -th root of $e(n)$ and the quotient $m(n)/n$ tend to limits as n tends to infinity. These limiting values are approximately 1.7070 and .5348 for the plane trees and approximately 1.7370 and .4901 for the labelled trees. Joint work with Professor A. Meir.

Some results on wreath products of graphs

Joy Morris

A variety of results will be presented on wreath products of graphs, focussing on their automorphism groups. Some results that have been achieved by using the wreath product as an inductive step will be mentioned. Much of this is based on joint work with Ted Dobson of Mississippi State University.

Contour Vertices and Their Applications to Convexity Notions in Graphs

Ortrud R. Oellermann

A vertex v of a graph G is a contour vertex if its eccentricity, $ecc(v) = \max\{d(v, u) | u \in V(G)\}$, is at least as large as that of its neighbors. The interval between a pair u, v of vertices in a graph G is the collection of all vertices that belong to some shortest $u - v$ path in G . A set S of vertices is convex if it contains the interval between all pairs of vertices in S . The convex hull of a set S of vertices in a graph G is the smallest convex set that contains S . We show, for any connected graph G and any convex set S of G , that S is the convex hull of the contour vertices of (the subgraph induced by) S . The closure of a set S of vertices of a graph is the union of intervals between pairs of vertices of S taken over all pairs of vertices of S . A set S of vertices of G is a geodetic set if its closure is $V(G)$. The smallest cardinality of a geodetic set for a graph G is called the geodetic number of G . We show that in general the problem of finding the geodetic number of a graph is NP-hard. For distance hereditary graphs it is shown that the contour vertices form a geodetic set. The Steiner interval of a set S of vertices in a graph is the union of all vertices that belong to some Steiner tree for S (i.e., a tree of smallest size that contains S). A set S of vertices in a graph G is a Steiner geodetic set if its Steiner interval is $V(G)$. The smallest cardinality of such a set is the Steiner geodetic number of the graph. We compare the geodetic and the Steiner geodetic number of a graph and show how contour vertices can be used to find the Steiner geodetic number for most distance hereditary graphs.

Two summer undergrads and their research

Bill Sands

I supervised two undergraduate students holding NSERC Research Awards this past summer. In this talk I will discuss a couple of combinatorial problems that I gave them to work on, and reveal the progress made on these problems.

Discussion of g_4^4

Ralph Stanton

The quantity g_4^4 is the size of the minimal pairwise balanced design using pairs, triples, and quads, such that every pair is covered exactly 4 times. This extends earlier work in which the lambda value for the covering was 1, or 2, or 3.