

**COLLOQUIUM**  
**University of Regina**  
**Department of Mathematics and Statistics**

**Speaker:** Peter Quast (University of Augsburg, Germany)

**Title:** Generalizing surfaces of constant mean curvature

**Date:** Friday, March 7, 2008

**Time:** 3:30

**Place:** Math & Stats Lounge (CW 307.20)

**Abstract**

The Gauss map of a (parameterized) surface assigns to each point the unit normal vector, which is an element of the 2-sphere. One may ask how geometric properties of a surface are encoded in its Gauss map. An example is the famous theorem of Ruh and Vilms. It states that a conformally parameterized surface has constant mean curvature (CMC) if and only if its Gauss map is harmonic (i.e. a critical point of the energy functional). On the other hand given a harmonic map into the 2-sphere one can (under suitable assumptions) construct a pair of conformal CMC-surfaces whose Gauss map is the given harmonic map. This construction goes back to Bonnet in the 19th century. At the beginning of the 1990s Bobenko gave a more explicit and geometric way to recover CMC surfaces from their harmonic Gauss maps.

After having reviewed this construction, we will discuss a generalization. Since the Bonnet-Bobenko construction relies on the way the usual 2-sphere sits in euclidean 3-space, we will first look at appropriate generalized target spaces. It turns out that hermitian symmetric spaces of compact type enjoy standard embeddings into euclidean spaces similar to the one of the sphere. A natural generalization of harmonic maps to complex manifolds of higher dimension are pluriharmonic maps, which are harmonic along every complex curve. Our generalized construction yields interesting submanifolds and provides a new relation between certain pluriharmonic maps.

Joint work with J.-H. Eschenburg.